Ondas não lineares chamadas solitões

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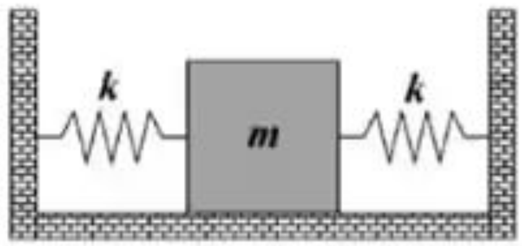
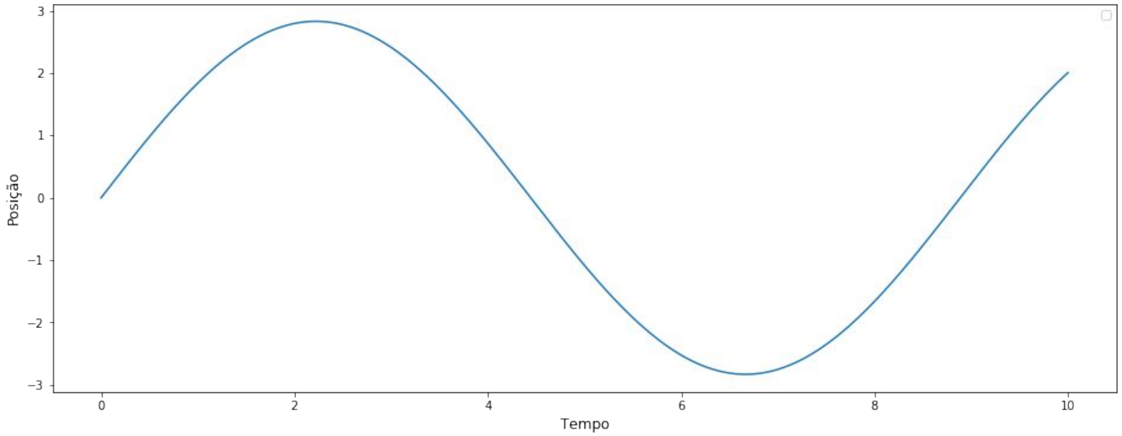
Duas molas, uma massa

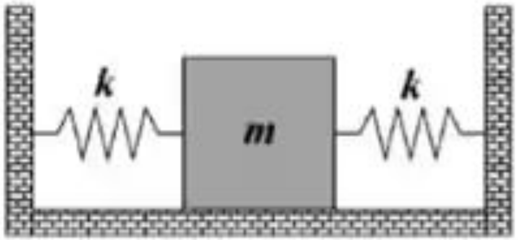


$$F(t) = -2kx(t) \implies a(t) = \frac{-2k}{m}x(t)$$

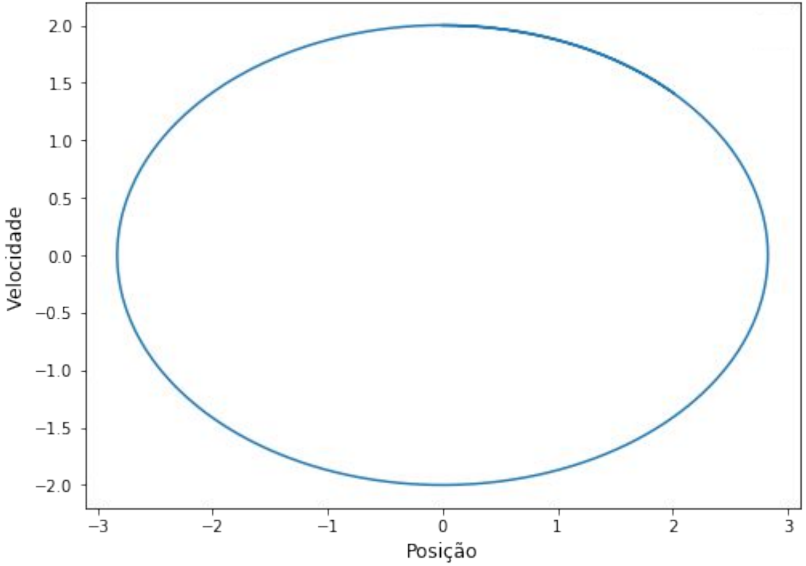


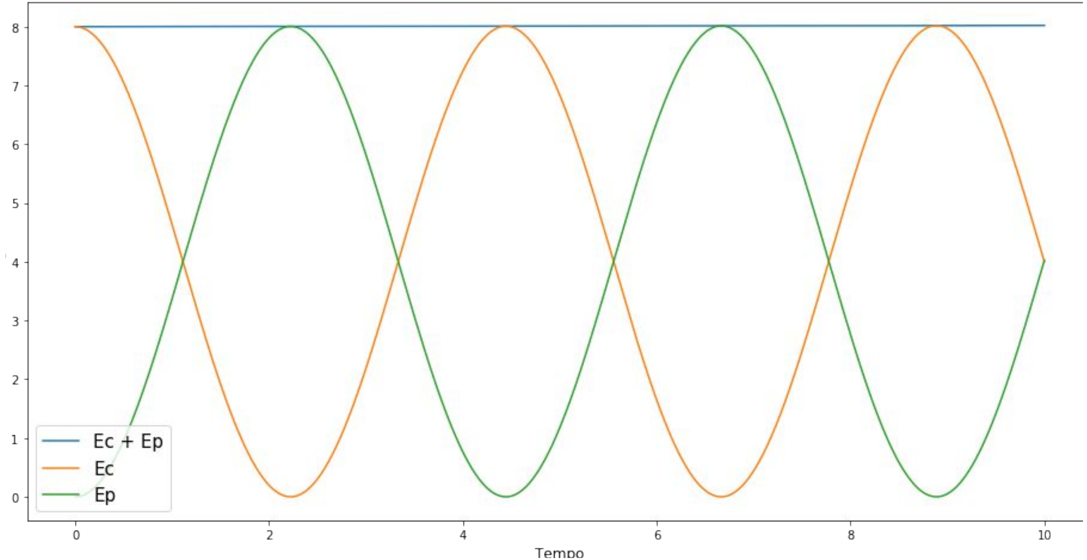
$$\begin{cases} v(t_{i+1}) = v(t_i) + a(t_i)\cdot\Delta t \\ x(t_{i+1}) = x(t_i) + v(t_i)\cdot\Delta t + \frac{1}{2}a(t_i)\cdot(\Delta t)^2 \end{cases}$$

Conservação de energia



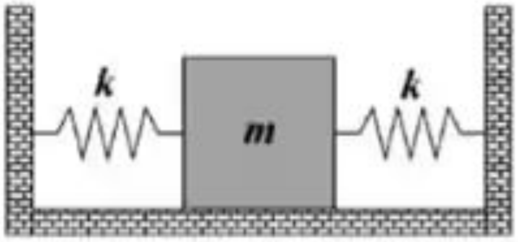
$$E_c = \frac{1}{2}mv^2 \qquad E_p = kx^2 \qquad \frac{E}{km} = \left(\frac{v}{\sqrt{2k}}\right)^2 + \left(\frac{x}{\sqrt{m}}\right)^2$$

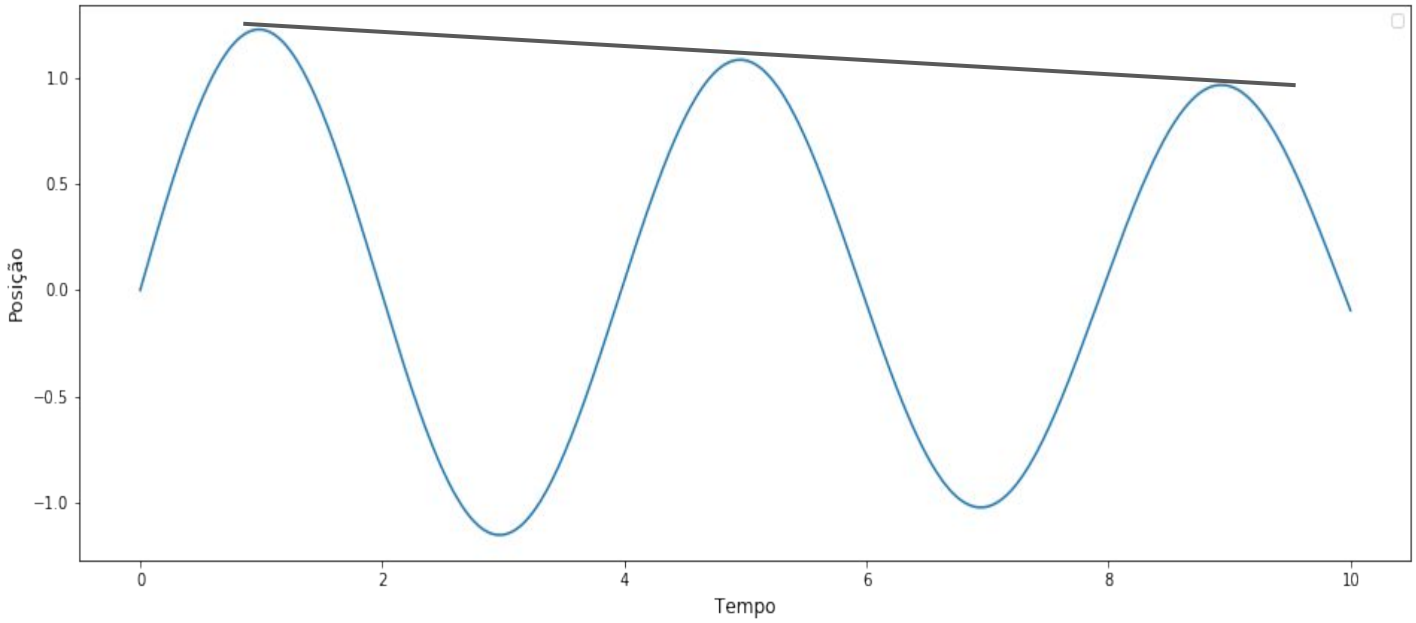




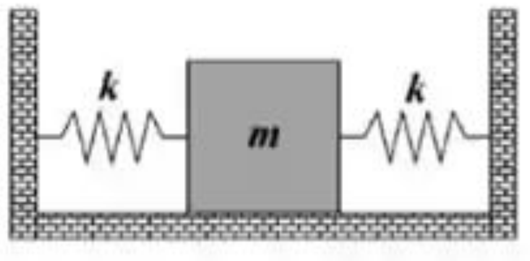
Dissipação de energia

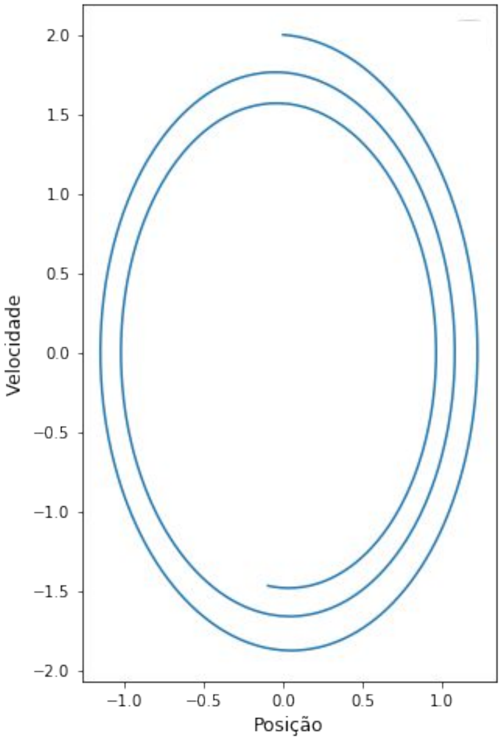
$$F = -2kx - \beta v - \lambda |v| v$$

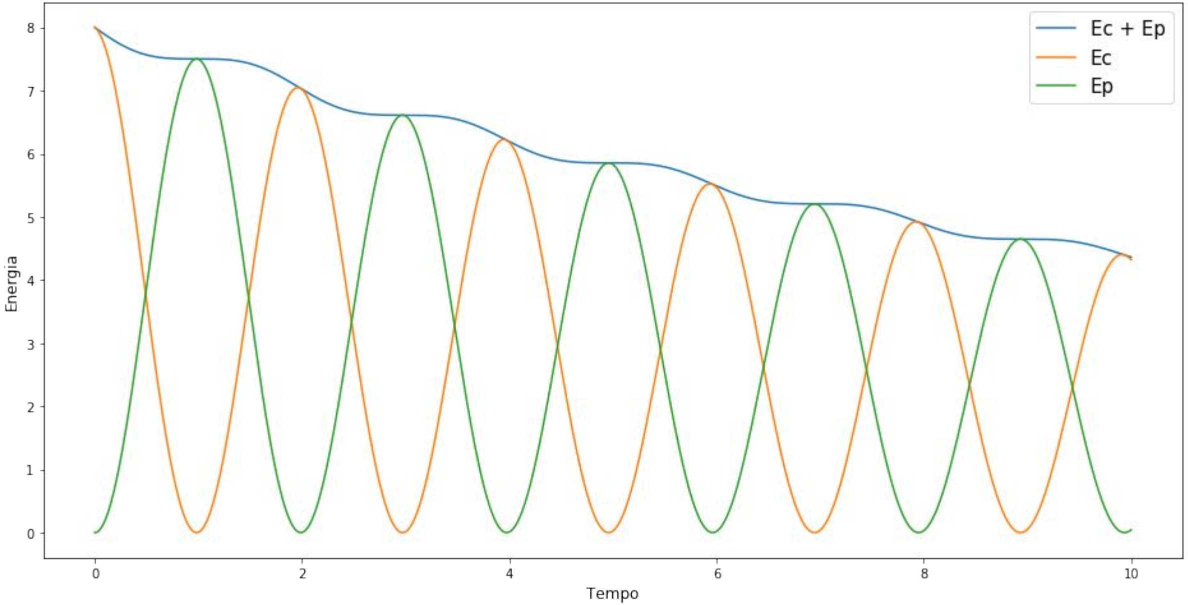




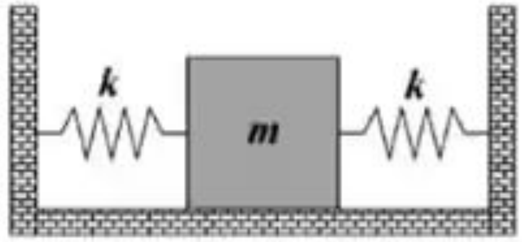
Dissipação de energia



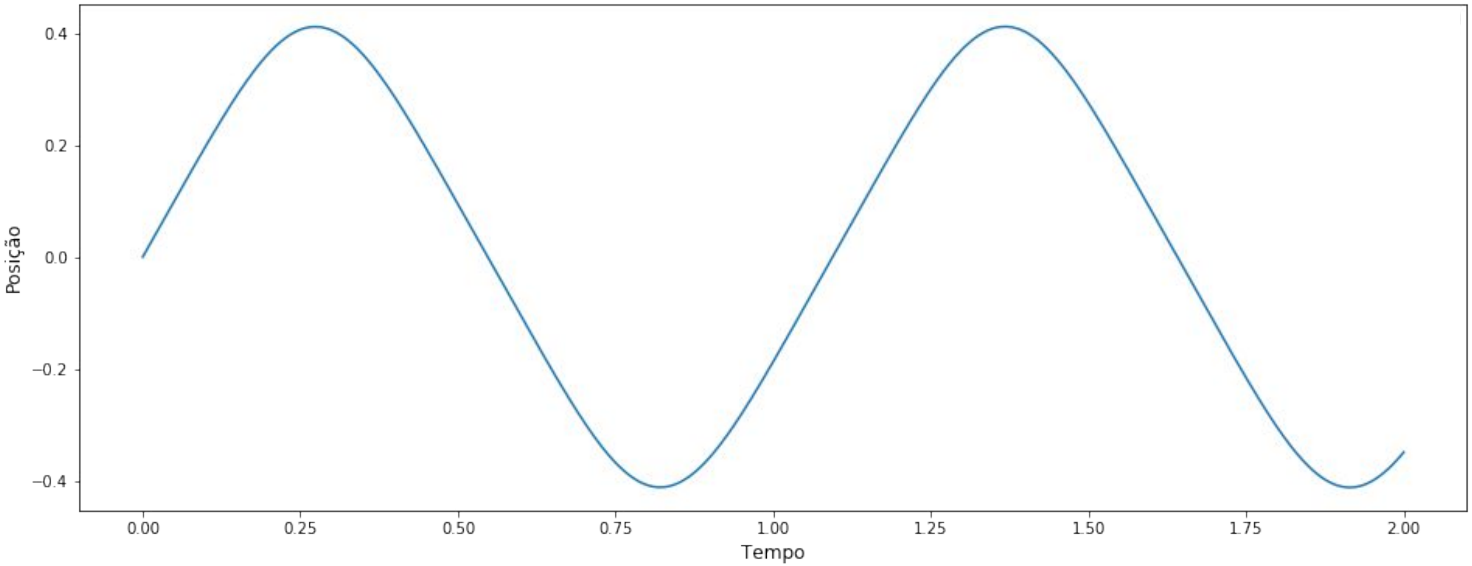




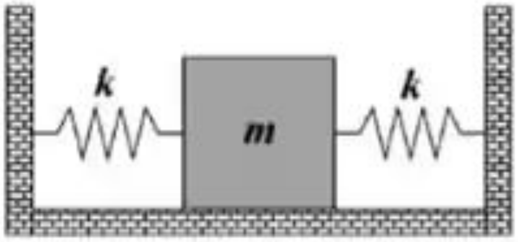
Não linearidade



$$F(x) = -2kx - 2bx^3$$

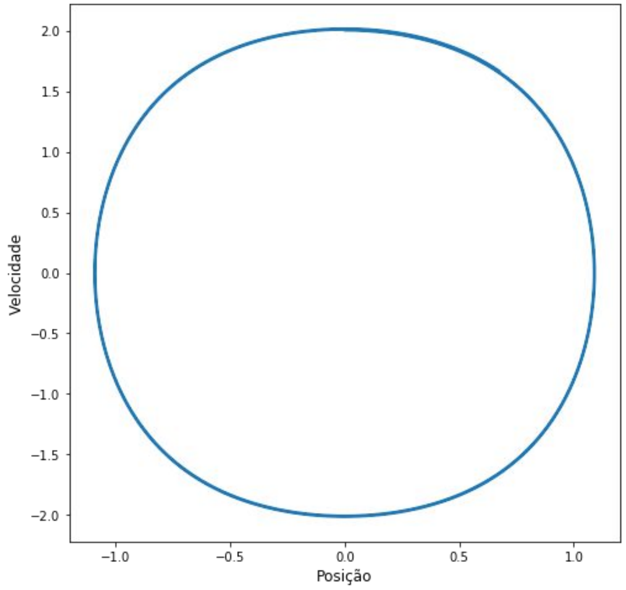


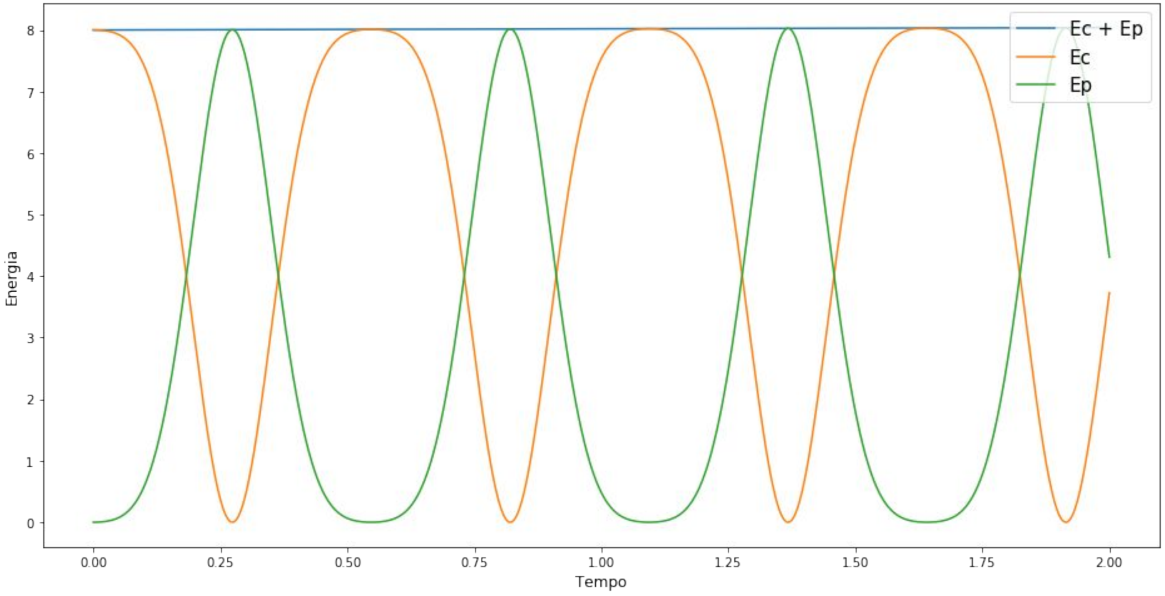
Não linearidade



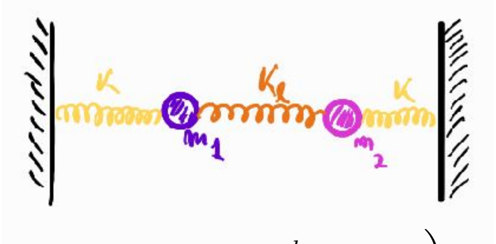
Há conservação de energia:

$$E_c = \frac{1}{2}mv^2 \qquad E_p = kx^2 + \frac{1}{2}bx^4$$



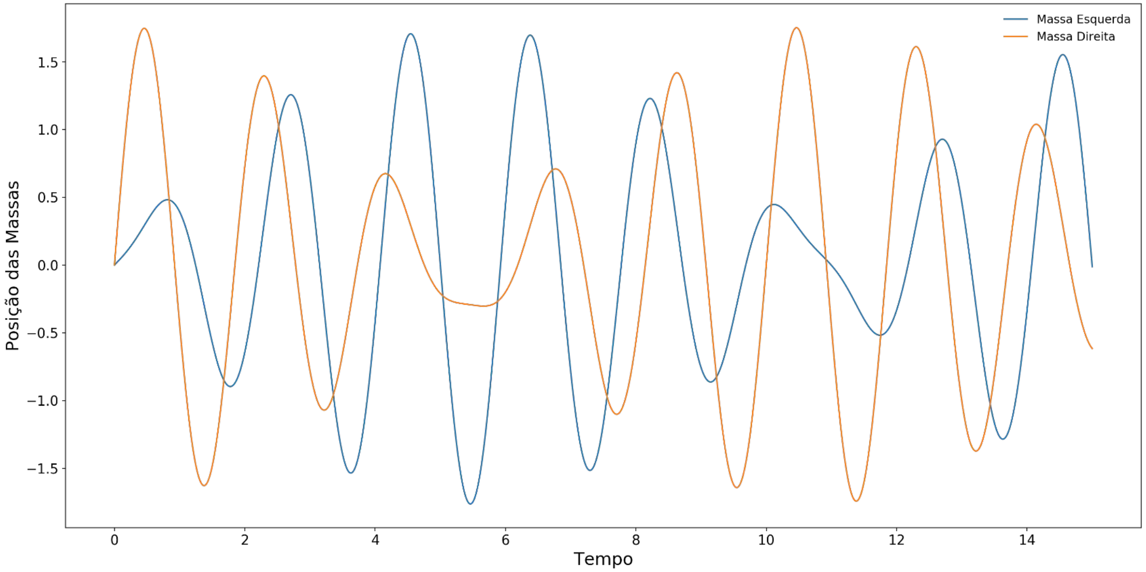


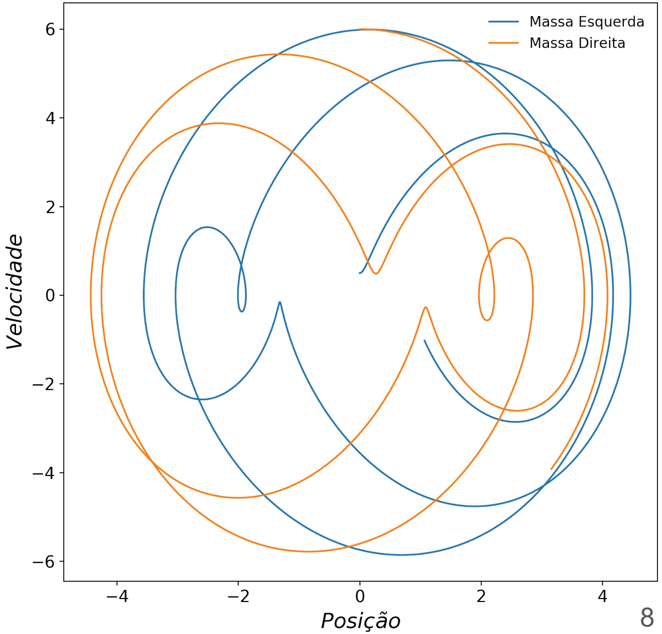
Três molas, duas massas



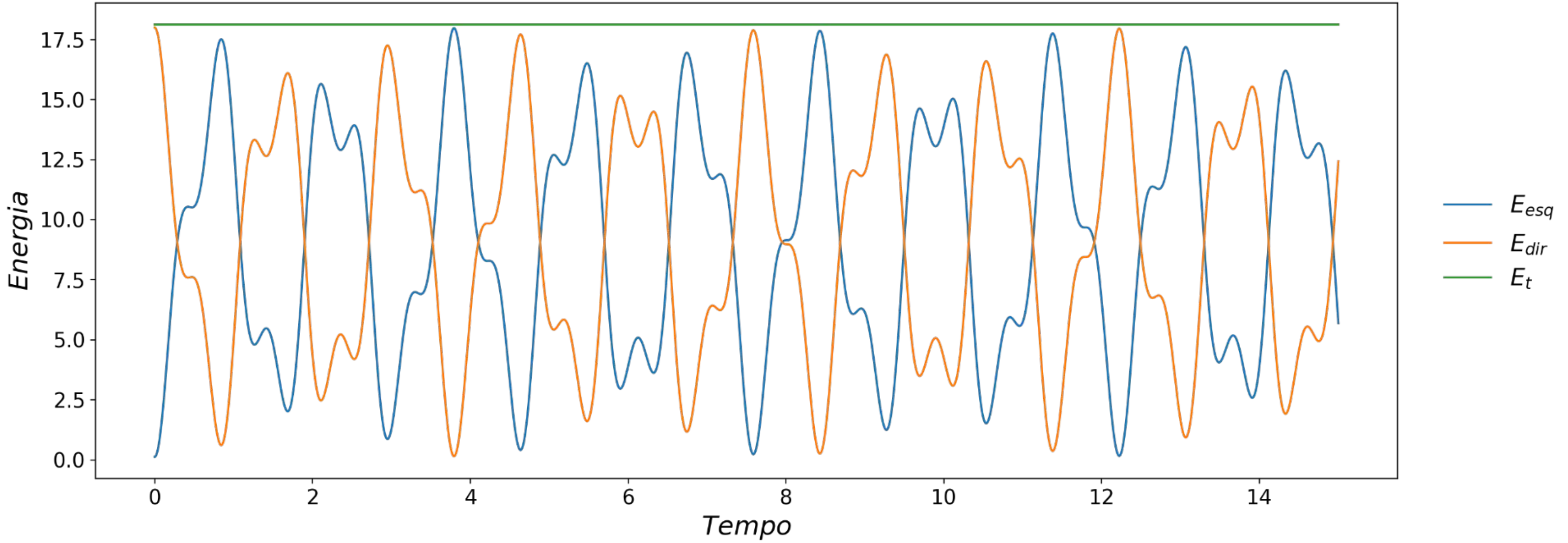
$$\begin{cases} F_{esq}(t) = -k(x_{esq}) + k_l(x_{dir} - x_{esq}) \\ F_{dir}(t) = -k(x_{dir}) - k_l(x_{dir} - x_{esq}) \end{cases}$$

$$\begin{cases} a_{esq}(t) = \dfrac{k_l}{m_{esq}}\left(x_{dir}(t) - x_{esq}(t) - \dfrac{k}{k_l}x_{esq}(t)\right) \\ a_{dir}(t) = \dfrac{k_l}{m_{dir}}\left(x_{esq}(t) - x_{dir}(t) - \dfrac{k}{k_l}x_{dir}(t)\right) \end{cases}$$

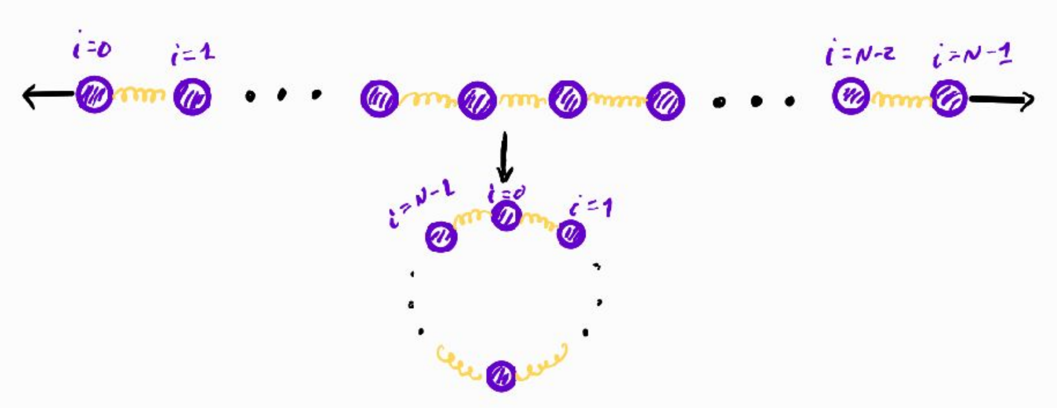




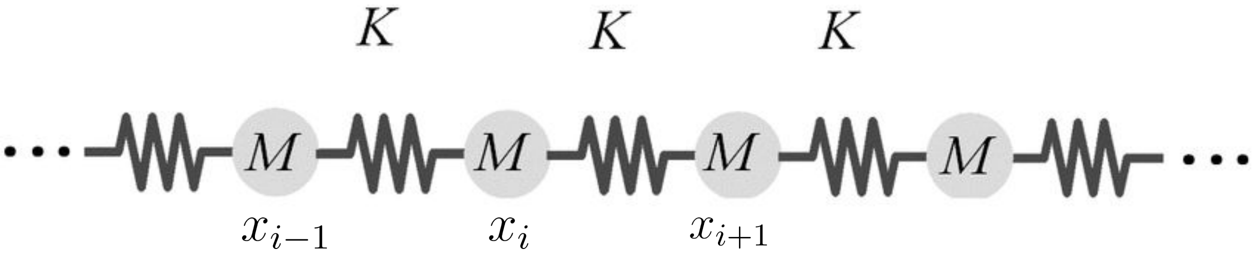
Energia, massa



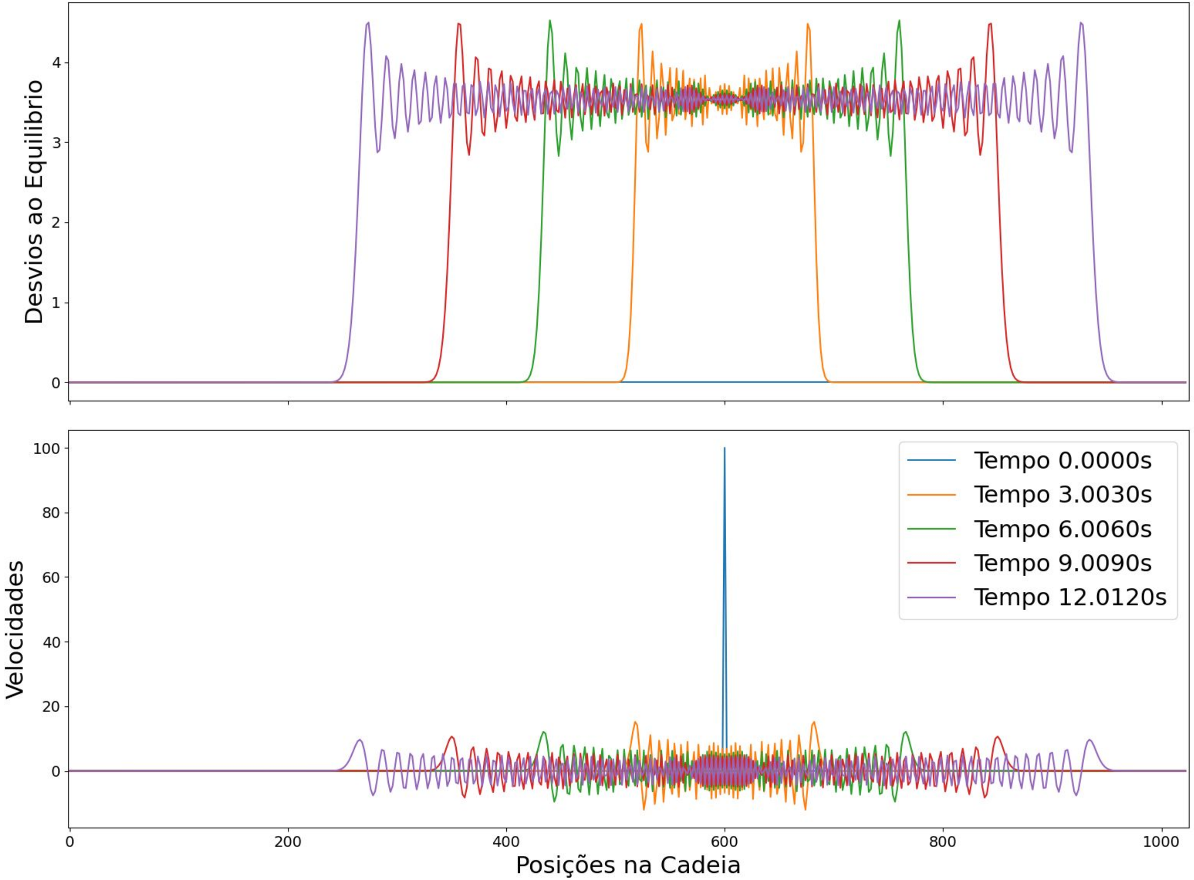
Cadeia periódica



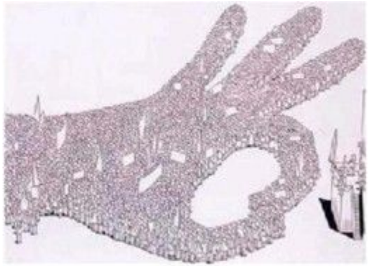
$$a_n(t) = \frac{k}{m} \cdot (x_{n+1} + x_{n-1} - 2\,x_n)$$



Piparote



Desvios ao Equilibrio
4
3
2
1
0
Velocidades
100
80
60
40
20
0
Tempo 0.0000s
Tempo 3.0030s
Tempo 6.0060s
Tempo 9.0090s
Tempo 12.0120s
0
200
400
600
800
1000
Posições na Cadeia



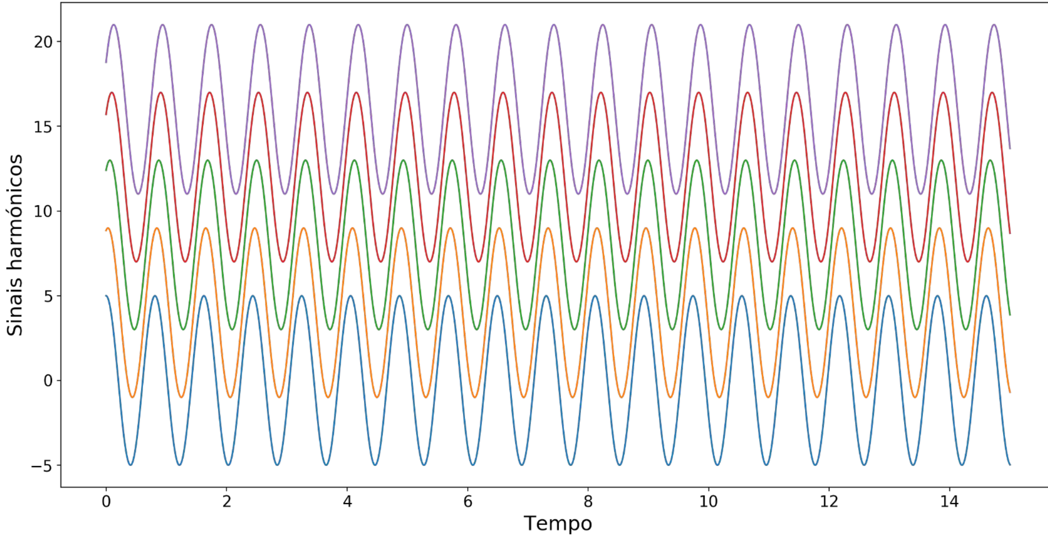
Onda harmónica

$$\begin{cases} x_i\left(t\right) = A\cos\left(p\Delta Di - \omega t\right) \\ a_i(t) = -A\omega^2 cos(p\Delta i - \omega t) = \frac{k}{m}(x_{i+1} + x_{i-1} - 2x_i) \end{cases}$$

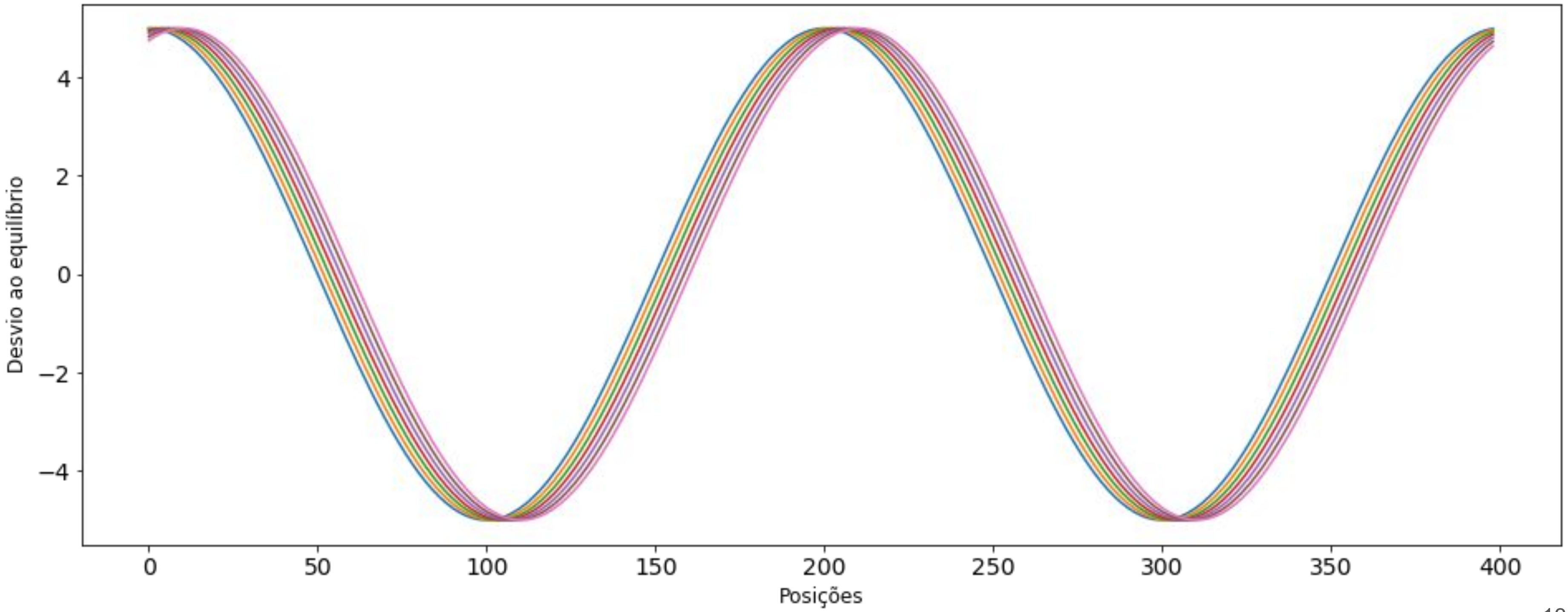
$$\Longrightarrow \ \omega\left(p\right) = 2\sqrt{\frac{k}{m}}\left|\sin\left(\frac{1}{2}p\Delta D\right)\right|$$

$$x_i = x_{i+N} \Longrightarrow p = \frac{2\pi}{\Delta DN}n, \ n \in \{1, \ldots, N\}$$

$$\omega = \frac{2\pi}{T} \qquad p = \frac{2\pi}{\lambda}$$



Onda harmónica

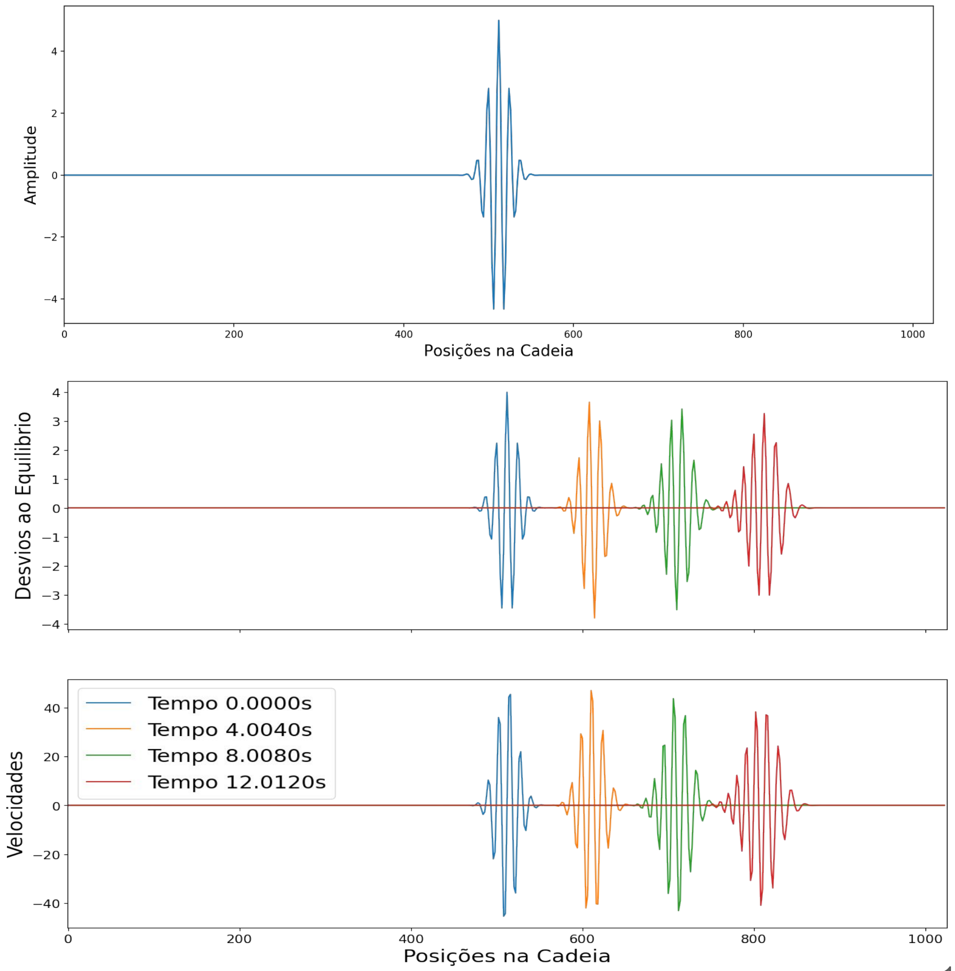


Pulso - Molas Lineares

O pulso é construído somando ondas harmónicas;

$$v\left(\lambda\right) = 2\lambda\sqrt{\frac{k}{m}}\left|\sin\left(\frac{\Delta D}{2\lambda}\right)\right|$$

Dispersão



Pulso - Molas não-lineares: “Solitão”

Introduzindo não-linearidade, contrariamos o efeito da dispersão;